Ellipse-approximated Isopach Maps for Estimating Ashfall Volume at Sakurajima Volcano

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In studies of volcanic tephra, it is usual that the overall volume of tephra is estimated ashfall volumes based on representative locations within the ashfall area. The precision of the volume estimation largely depends on the number of the locations. However, in the case of ongoing eruptions on island volcanoes, such as Sakurajima volcano, the observation locations are usually limited. We therefore have developed a practical method for estimating ashfall volume and distribution in such case. The method approximates the distribution of ashfall as ellipses, with the distribution area \( A \) and thickness or weight of deposit \( T \) determined by \( A = aT^{-1} \). The ellipse-approximated isopachs can be determined by using the direction of the ellipse axis and ashfall data at two points. In determining the ellipse axis exactly, we usually need additional ashfall amounts from the other locations. We set 37 samplers around Sakurajima volcano, and retrieved the samplers 15 times, from April to December, 2008. Using the propose method, we are able to determine the volume of ash produced by small, continuous eruptions.

**Key words**: ashfall distribution; isopach; ellipse approximation; volume estimation; Sakurajima volcano

1. Introduction

The difficulty of ashfall volume estimation for small islands or short intermittent eruptions in Japan is a major issue, when forecasting the progress of volcanic eruptions. Hence, we propose a tool for estimating ashfall volume using limited ashfall observation locations as inputs. Such tool is useful in the case of ongoing eruptions when observation locations are usually limited.

Many authors have proposed methods to calculate the distribution or volume of pyroclastic falls. The most advanced method is to produce an isopach map (lines of equal thickness) of tephra deposits, based on many points determined by geological surveys. Other approaches to volumetric calculations include simulation models based on the dynamics of the eruption column and existing models or codes (Ishimine, 2007). An isopach is a contour line that shows the same thickness of tephra for a given area. Whole ashfall volume can be estimated using isopach map by applying integral calculus. Many studies have shown that tephra thickness decreases exponentially with distance from the vent. Thorarinsson (1954) showed important results related to the exponential decrease in tephra thickness with increasing distance from the source. Porter (1973) considered that the correlation of thickness and distance followed a power relationship for Hawaiian tephra. Suzuki (1981) presented a logarithm approximation method that explained these correlations.

If the area and tephra thickness are correlated, then the volume of tephra can be deduced. Rose et al. (1973) estimated ashfall volume by integrating the plot of \( \log(T) \) against \( \log(A) \) with distribution area \( A \) and thickness or weight of deposit \( T \). Recently, ashfall deposits were shown to follow exponential decreases of \( \log(T) \) against \( \log(A) \) for plinian tephra (Pyle, 1989). Fierstein and Nathenson (1992) used two proximal and distal exponential rates \( (\kappa) \) of the plot of \( \log(T) \) against \( \log(A) \), which changed at the break in slope, in order to calculate the volume. Bonadonna and Houghton (2005) used a power method to estimate the volume of Plinian tephra deposits. When using the exponential method or the power formula, many isopachs are required to estimate the ashfall volume over large areas accurately.

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Aramaki and Hayakawa (1982) developed a simple formula for the plot of log(T) against log(A), with a scaling exponent (power) fixed at −1. Hayakawa (1985) estimated the ejecta volume as $V = 12.2 \times T \times A$ ($V$: volume of tephra, $A$: area, $T$: thickness of tephra) with a constant of 12.2. This value of 12.2 was in agreement with results obtained from the crystal concentration method (Walker, 1980; 1981), in which the ejecta volume was estimated by the amount of plinian tephra observed on the ground. However, Pyle (1999) showed that the value of 12.2 was variable rather than constant.

We consider the geometrical concept of ashfall distribution, which does not require the collection or estimation of ashfall distribution or volume over large areas. For example, small island volcanoes require a volume estimate only for the limited area around the volcano. Recently, some authors considered that ashfall distribution would approximate the elliptical forms (Froggatt, 1982; Pyle, 1989; Sulpizio, 2005). Sulpizio (2005) calculated the approximate volume of tephra using elliptical distributions. In this study, we have developed an ellipse-approximated isopach (EAI) method based on a simple equation. We have applied this method to the volume

Fig. 1. Location of the Sakurajima volcano. Triangles denote active volcanoes.

Fig. 2. Observation locations in Sakurajima. Solid circles denote locations of ash samplers for this study. Open circles denote monthly observation points and camera marks used by the Osumi Office of River and National Highway.
estimation of ashfall for ongoing eruptions of Sakurajima volcano (Figs. 1 and 2), which have produced frequent thermal columns from Vulcanian or ash eruptions. The depositional ashfall (tephra) volume and the ejecta ashfall (tephra) volume of an eruption are different meaning (Koyaguchi, 1996). In this study, we use the word “ashfall” to mean “depositional ashfall”.

2. Ellipse-approximated isopach (EAI) method

2-1 Area-versus-thickness relationship

We use the relationship of the simple power function between log(T) and log(A) (see Eq. 1), where A is the area of one isopach, T is the thickness for that isopach, α is a coefficient and d is the exponent concerning T and A shown in equation (1). In this study, area (A) is given in square meters and thickness (T) in millimeters or grams per square meter.

\[ A = \alpha T^d \]  

It is uncertain whether the concept (d = −1) proposed by Aramaki and Hayakawa (1982) is appropriate for small phreatic, Vulcanian or ash eruptions. We therefore review the relationship between ashfall thickness and area from recent Vulcanian and phreatic eruptions in Japan. We use the described isopach maps to plot depositional area (m²) versus thickness (mm). In this case, we convert g/m² to m, using a depositional density of 1.5 g/cm³ for the data produced by Yoshimoto et al. (2005) and Takarada et al. (2001).

At Shinmoedake volcano in the Kirishima volcanoes, southern Japan, phreatic eruptions started at 14:50 JST on February 17th, 1959, and continued for several days. An isopach map was obtained for this eruption (Fig. 3; Geshi et al., 2010). These results show that the area-versus-thickness relationship is A = \alpha T^{-1}, and that the phreatic and magmatic ashfall eruptions show a rate of same decrease (d = −1) in Fig. 3. In addition, the 1959-Shinmoedake, 1979-Ontake and 2008-Shinmoedake eruptions were comprised of multiple eruptions or a continuous eruption; those cases also showed a rate of decrease (d = −1). Therefore, we adopt a simple formula with the scaling exponent (power) fixed at approximately −1, as described by Aramaki and Hayakawa (1982). Regarding shapes of isopach, we assume that the tephra distribution approximates an ellipse that has the same aspect ratio (half radius of orthogonal axis/half radius of calculation axis) in concurrent eruptions (Fig. 4) with the correlation of A and T, following Eq. 1.

2-2 Formulation of EAI and the volume

The followings are three calculation ways of the ellipse-approximated isopach (EAI).

1) If the elliptical isopachs exhibit a fixed aspect ratio, we can calculate the ellipse-approximated isopach using one data point of the thickness or weight of the deposit and the determined calculation axis (one data point calculation).

2) If we do not know the aspect ratio of the elliptical isopach, we can calculate the ellipse-approximated isopach using two data points of the thickness or weight, and the
determined calculation axis (two data points calculation).

3) If the ellipse isopach does not exhibit an aspect ratio or an ellipse axis, we can calculate the ellipse-approximated isopach using three or more data points of the thickness or weight (multiple data points calculation).

Solution using one data point:
The elliptical isopach exhibits a fixed aspect ratio, defined as
\[ \frac{b}{a} = c \] (2)
where \( a \) is the calculation (ashfall distribution) axis of the ellipse; \( b \), the orthogonal axis; and \( c \), the aspect ratio.

A point \((x, y)\) on the ellipse is given by
\[ \frac{(x-a)^2}{a^2} + \frac{y^2}{b^2} = 1 \] (3)

The ellipse axes are determined from Eqs. 2 and 3 are
\[ a = \frac{c^2}{2c^2} \quad b = a \times c \] (4)

Solution using two data points:
If ellipses 1 and 2 have the similar ellipse shapes (Fig. 4), then
\[ \frac{b_1}{a_1} = \frac{b_2}{a_2} \] (5)

From Eq. 1, \( AT = a \), and \( A_1 = \pi a_1 b_1 \) and \( A_2 = \pi a_2 b_2 \), The relationship between ellipses 1 and 2 is
\[ T_1 a_1 b_1 = T_2 a_2 b_2 \] (6)

From Eqs. 5 and 6,
\[ \frac{a_1}{a_2} = \frac{T_1}{T_2} \] (7)

\( P_0 \) is source of ash distribution. If \( P_2 \) on ellipse 2 moves to a point on ellipse 1 using similar triangles shown by dash lines in Fig. 4, then
\[ P_1 = (x_1, y_1), \quad P_2 = \left( \sqrt{\frac{T_1}{T_2}} x_2, \sqrt{\frac{T_1}{T_2}} y_2 \right), \quad P_0 = (0, 0) \] (8)

\( P_1 \) and \( P_2' \) on ellipse 1 are given by:
\[ \frac{(x_1-a)^2}{a_1^2} + \frac{y_1^2}{b_1^2} = 1 \] (9)
\[ \left( \frac{T_1}{T_2} x_2 - a_1 \right)^2 + \left( \sqrt{\frac{T_1}{T_2}} y_2 \right)^2 = 1 \] (10)

The ellipse axes are determined from Eqs. 9 and 10 as follows:
\[ a_1 = \frac{x_1^2 y_1^2 - x_1 y_1^2}{2 x_1 y_1 \sqrt{T_1 T_2}}, \quad b_1 = \sqrt{\frac{a_1^2 y_1^2}{2 x_1 y_1}} \] (11)

Solution using three data points:
If we consider three or more data points, any of the three approaches may be used to derive the solution. The three formulae are determined about the calculation axis (a), orthogonal axis (b), and calculation axis at a specified angle (θ). Here we show only the basic formulae.

We consider ellipses 1, 2, and 3, which have similar shapes:

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**Fig. 4.** Schematic representation of isopach drawn by ellipse approximation. The figure shows the EAI method for two data points. The x-axis represents the fixed dispersion (calculation) axis.
Similarly to Eq. 6, 
\[ T_1 a_1 b_1 = T_2 a_2 b_2 = T_3 a_3 b_3. \] (13)

The three points move according to a rotation matrix, for example:
\[
\begin{pmatrix}
  x' \\
  y'
\end{pmatrix} = 
\begin{pmatrix}
  x_1 \cos \theta - y_1 \sin \theta \\
  x_1 \sin \theta + y_1 \cos \theta
\end{pmatrix} = 
\begin{pmatrix}
  x_2 \cos \theta - y_2 \sin \theta \\
  x_2 \sin \theta + y_2 \cos \theta
\end{pmatrix} \quad (14)
\]

Eq. 14 is substituted into Eq. 11 and the two equations are solved numerically. Under natural conditions, it is impossible that three data points fit on an ellipse of one aspect at the same time.

The ashfall volume is calculated using the EAI as follows, using the distribution \( A = \alpha T^{-1} \). The volume integral is:

\[
V = \int^m \! \! \text{AdT} = (-\alpha \log(A_o)) - (-\alpha \log(A_n)) \quad (15)
\]

where \( m \) is the 10^4 m² area used by Takarada et al. (2001) and \( n \) is the area enclosed by the 0.1 g/m² isopach. The minimum observed thickness corresponded to a weight of 0.4 to 0.2 g/m² for the 12:56 JST eruption on April 28th at Sakurajima volcano (Fig. 5). Our field observations determined the lower threshold of detectable ashfall to be 0.1 g/m².

In this study, we discuss how to calculate for EAI (eq. 11) and estimate for the volume of tephra (eq. 15) using the two data points calculation.

### 2-3 Relationship between ellipse axis and volume of EAI

In this section, we demonstrate how to use the EAI method namely how to choose two data locations and set an ellipse axis. In the two data points calculation, it is very important to determine the calculation axis of the ellipse accurately. A trial calculation uses two data points for tentative ashfall amounts. The tentative amounts are set at sampling locations HR1 (100 g/m²) and AR4 (50 g/m²) at Sakurajima volcano (Fig. 7). Those are a value commonly observed at Sakurajima. The calculation results include ashfall volume and the aspect ratio of the ellipse, which is rotated from the calculation axis of the ellipse to one degree clockwise from due east (Fig. 6).

The axis of the ellipse is regarded as non-existent on the
opposite side of the observation points ($171^\circ < \theta < 4^\circ$).

Firstly, the calculation axis of the ellipse is calculated to be four degrees clockwise from east. In this case, the aspect ratio is greater than one, meaning that the calculation axis is the short axis and the orthogonal axis is the long axis. The EAI distribution is a long orthogonal axis, which is an unusual result under natural conditions. The results are the same from $4^\circ$ until $24^\circ$ clockwise from due east (Fig. 7a). A solution cannot be got past $25^\circ$ ($24^\circ < \theta \leq 87^\circ$), because the position of a low amount ($50$ g/m$^2$) interchanges a high amount ($100$ g/m$^2$) position for calculation axis in this area.

We must be careful in the case when the calculation axis is close to the observation point, which is a common case for the EAI calculation. The slight difference in the angle of the calculation axis leads to a larger change in the aspect ratio and the ashfall volume (Fig. 6). If the interval of the angle is very small, the infinite volume is taken between $87^\circ$ and $88^\circ$, which results in an extremely elongated ellipse (Fig. 7b). It is considered that the narrow lateral distribution of ashfall is due to a very strong wind over a long period. The minimum ashfall volume occurred along $90^\circ$.

Fig. 7. The schematic representation of the results of the trial calculations by the EAI method. We used the virtual ashfall amount measured at two points ($100$ g/m$^2$ and $50$ g/m$^2$). The calculation axis is rotated clockwise from east to north. The shaded area shows where we cannot use ellipse approximation to obtain a solution. A solution is obtained for the white area by ellipse approximation. In this case, we obtained the true circle solution at the dot-dashed line labeled “$b/a = 1.0$”. 

$(a) 4 \leq \theta \leq 24$

$(b) 87 \leq \theta < 90$

$(c) \theta = 90$

$(d) 90 < \theta < 151$

$(e) 151 \leq \theta \leq 171$
We applied our proposed method on the ashfall around Sakurajima volcano. Ash samplers, which consisted of
clear plastic cups of 7 to 8 cm diameter, were placed at three locations between February and April 2008. We tested the measurement error for this cup method at the southern and east part of the volcano set 13 samplers, and found that the observation errors were less than 4%. After the increase in activity from April 2008, we placed additional ash samplers at 37 locations around the volcano on April 24th and 25th (Fig. 2). Ashfall deposits from the samplers were collected at different time intervals ranging from days to several weeks. The dates of retrieval were April 27th, May 1st, 7th, 9th, 18th, and 29th, June 5th and 14th, July 4th and 12th, August 18th and 30th, September 23rd, and October 18th. The ashfall characteristics were noted at the sampler locations in the field, after which the samplers were carefully covered with clear plastic cling-wrap and transferred to the laboratory. We soaked clumps of ash in distilled water to separate out smaller flakes. The deposit samples were dried and measured in terms of grams per square meter to determine the isopach. We converted g/m² to mm by applying the depositional density of the fresh ashfall deposit after the April 3rd - 7th eruptions at sampling point AR1. We placed the ashfall deposits into a mould in order to undisturb the samples, which were then used to measure the depositional density under dry conditions. The resulting average depositional density is 1.5 g/cm³ (Table 2), showing that an ashfall depth of 1 mm is equivalent to 1500 g/m².

### 3-3 The determination processes of ashfall axis

During observation periods, we determined the approximate directions of ashfall dispersion as follows (Table 3). Occasionally, observers remained at the volcano and noted

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Density of ashfall deposits under dry condition (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AD-1</td>
<td>1.39</td>
</tr>
<tr>
<td>AD-2</td>
<td>1.63</td>
</tr>
<tr>
<td>AD-3</td>
<td>1.48</td>
</tr>
<tr>
<td>Average</td>
<td>1.50</td>
</tr>
</tbody>
</table>

Table 2. Density of ashfall deposited by eruptions between Feb. 3rd and 7th, 2008.
the direction of the column. When we could not identify the distribution axis, we used web-cameras and other information. Our first source of eruption information was the website of the Japan Meteorological Agency1) (JMA), which immediately releases eruption information, categorized by eruption time, column height, column direction, intensity, and other parameters. Next, we used images from web-cameras set up at the Osumi Office for River and National Highway of the Ministry of Land, Infrastructure, Transport and Tourism (MLIT)2), and in the city of Tarumizu3). Kagoshima University (KU) also provided archives of their web-camera images4), which yielded valuable information for determining the column direction (axis).

The determined ashfall directions described above usually have some uncertainties. Followings are the general procedure to determine the calculation axis exactly. If an eruption occurred once during an observation period, we chose generally the maximum observed amount (weight or thickness) at first near the expected distribution axis and next the second or third-largest value for the two data points of the EAI calculation. Next, we rotate an EAI calculation axis by step intervals of 1 to 2 degrees around the expected direction (we call this procedure as the step seeking hereafter) and calculate EAI distribution repeatedly every degrees. We can calculate the ashfall amounts of nearby locations, where the actual amounts of ashfall are measured using the EAI method and Eq. 16. Finally, we determined the EAI calculation axis so that the differences between the calculated and measured ashfall amounts become minimum value. The determination process is more easily when the exact direction is determined by the observation.

3-4 An example of general determination of EAI; June 28th, 2008 eruption

We explain how to determine the ashfall volumes for one eruption recorded at 06:36 JST on June 28th along with small eruptions on the afternoons of June 28th and 29th (JMA website). The ashfall of the small eruptions would be negligible in amount. We retrieved the samplers on July 5th. Ashfall was mainly found in the northern areas of the volcano. Ash weights of up to 263, 203 and 281 g/m² were recorded at sampling locations AM, FK1 and MU, respectively (Fig. 9). This ashfall originated from Showa crater after an eruption at 06:36 JST on June 28th. Based on the amount of ashfall, the distribution axis would pass through somewhere between locations AM and FK1. The angle of the EAI calculation axis was determined by the step seeking so that the calculation agreed with the weight recorded at other locations (KM2, KM1, KT1 and KT2) and limit of ashfall distribution (Table 3, calculation No. C58). The calculation axis was 275° clockwise from due east, and an EAI was drawn based on the data of locations AM and FK1 (Fig. 9). The location names and ashfall amounts of basic two data points and the appropriate ones are listed in Table 3. The estimated numerical values of the degree of axis are also listed.

3-5 An example in the case of the multiple eruptions; May 2nd to 7th, 2008 eruptions

When the interval of eruptions is shorter than that of retrieval of samples, it is very difficult to estimate the amount of each ashfall. We explain how to determine the ashfall volumes in the case of May 2nd to 7th observation period. During this period, four eruptions were recorded, at 06:34–06:50, 15:29, and 16:05–16:30 JST on May 6th and 06:38–06:54 JST on May 7th. Further smaller eruptions with negligible ashfall amounts also occurred on May 6th and May 7th (JMA website). During this period, the ashfall samples were retrieved once. Based on the amounts of the ashfall samples, mainly four distribution axes could be observed from the Showa crater towards locations HM, KG, ST and HR1 (Fig. 10). The observed ashfall informations suggested that these axes were corresponded to those of 06:34–06:50, 15:29, and 16:05–16:30 JST on May 6th, and 06:38–06:54 JST on May 7th eruptions, respectively (Fig. 10). The EAI s for these four eruptions were drawn, based on two data points calculation located near the corresponding axis with the step seeking using appropriate data points. Some locations used for the calculations were affected by ashfall of other eruptions, so that the affected weight must be taken away from the weights collected at locations (Table 3, calculation No.C13 to No.C18). We could use the estimation methods based on Eq. 16, as explained in 2-4.

3-6 The exceptional cases; April 11th, May 8th and June 1st

When eruptions continue steadily for a long time and the wind direction changes gradually, an obvious distribution axis cannot be determined for such eruptions. In this case, we estimate the ashfall volume by isopach area drawn by hand. We examine the relationship of \( a = TA \) (\( a \): coefficient; \( T \): m, \( A \): m²) from Eq. 1 to the volume of the deposits. For example eruptions continued for over 2 hours, from 14:13–16:45 JST, on May 8th (JMA website) and ash emission continued for some hours after 16:45 JST. The ashfall was mainly found in the northeast to north areas, and in the lower amounts in the northwest to south areas of the volcano. First, we determined four isopachs from the northeast to north by EAI (Table 3, calculation No.C19 to No.C22). In the western part, we produced the 30 g/m² isopach by hand. The ashfall volume of 11330 t on May 8th (Table 3) was estimated from the relation shown in Fig. 11. However, the main application for this relationship is limited only valid for volumes less than 100,000 tons.
Table 3. List of Sakurajima eruptions from Feb. to Nov. 2008 and the results of the EAI volume calculation.

<table>
<thead>
<tr>
<th>Date (MM/DD)</th>
<th>Time (HH:MM)</th>
<th>Eruption Center</th>
<th>Collection Times (mm)</th>
<th>EAI Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>02/13/2008</td>
<td>08:40</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>02/13/2008</td>
<td>09:00</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>02/13/2008</td>
<td>09:30</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>02/13/2008</td>
<td>10:00</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>02/13/2008</td>
<td>10:30</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>02/13/2008</td>
<td>11:00</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>02/13/2008</td>
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<tr>
<td>02/13/2008</td>
<td>12:30</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The day and time and eruption center are from the JAM website. Unknown. Thick horizontal lines show collection times for ash fall samples. Multiple EAI distributions for one eruption were by continuously eruption depending on height. Locations (loc.) show in Fig. 2.
<table>
<thead>
<tr>
<th>Seeding point 1</th>
<th>Seeding point 2</th>
<th>Seeding point 3</th>
<th>Seeding point 4</th>
<th>Determined Al (°)</th>
<th>Aspect Ratio</th>
<th>Calculated Volumes</th>
</tr>
</thead>
<tbody>
<tr>
<td>loc.</td>
<td>obs.</td>
<td>seeking</td>
<td>loc.</td>
<td>obs.</td>
<td>seeking</td>
<td>loc.</td>
</tr>
<tr>
<td>(g.m²)</td>
<td>(g.m²)</td>
<td></td>
<td>(g.m²)</td>
<td>(g.m²)</td>
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<td>(g.m²)</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0.07</td>
<td>12.4</td>
<td>2</td>
<td>0</td>
<td>8.3</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>0.08</td>
<td>16.4</td>
<td>2</td>
<td>0</td>
<td>12.2</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>0</td>
<td>8.3</td>
<td>2</td>
<td>0</td>
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</tr>
<tr>
<td>8</td>
<td>2</td>
<td>0</td>
<td>16.4</td>
<td>2</td>
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<td>9</td>
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<td>16</td>
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<td>0</td>
<td>16.4</td>
<td>2</td>
<td>0</td>
<td>12.2</td>
</tr>
<tr>
<td>17</td>
<td>2</td>
<td>0</td>
<td>8.3</td>
<td>2</td>
<td>0</td>
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The "obs." values are observation weight. The subtracted values (s.v.) are calculated using Eq.16. The “calc.” values are weight for two data points calculation (calc. = obs. – s.v.). The “seeking” values are calculated weight on seeking point using Eq. 16. w, weak ashfall; x, no ashfall; c: other EAI calculation point.
In the cases of April 11th and June 1st, we calculated the ashfall using only one observation location. In these cases, we estimated the ashfall volume by the one data point calculation with the averaged aspect ratio of EAI. The aspect ratio of 0.21 was taken from average value in Table 3. The ashfall volumes were calculated to be 26990 t for April 11th and 1270 t for June 1st were estimated (Table 3, calculation No.C03 and No.C47).

4. Discussion

4-1 Comparison ashfall volume between EAI estimations and monthly MLIT observations

The Osumi Office of the MLIT, using 0.57 m-diameter drum-type samplers, measured the weight of monthly ashfall in several observation locations (Fig. 2). To validate our method, we compare the observed monthly weights obtained by the Osumi Office with the accumulate weight derived using the EAI distributions. We use Eq. 16 to calculate the tephra thickness for eruptions at the six observation locations of the Osumi Office shown in Table 3.
4. The data from the six observed locations show a positive correlation with our calculations (Fig. 12, Table 4). The observed values tend to be higher than the calculated ones for some data points, due to the intervals between observation locations. We note that the slightly higher observation values would be caused sometimes by contamination of the secondary ash.

4-2 Temporal variation of ashfall volume estimated by EAI method during 2008 activity

Using combined eruption informations and our observations at Sakurajima, we were able to determine the 68 directions of EAI dispersion and estimated 58 (66) eruption volumes (Table 3).

1) Volume for JMA classifications

According to the JMA classification scheme, the scale of the eruptions covers seven categories (Q1–Q7). These categories are determined by the area size (m²) of the eruption column by eye-watching observation. (Japan Meteorological Agency, 1994). Based on our EAI data, the average volume produced in the Q3 (medium) classification, which is most common at Sakurajima volcano, is about 5000 t, whereas the average volume produced in the Q4 (rather much) classification is about 20000 t in this study. Thus it is resonable to consider that the volume is divided according to the classification of JMA as Q3 : Q4 = 1 : 4 (Table 3).

2) Temporal change of ashfall volume during 2008 activity

During 2008, the first volcanic activity occurred from February 3rd to 7th. We estimated the ashfall volume of each eruption between February and October 2008, using the EAI method which the exhaustively axis determined by geological observation or JMA information (Table 3). The daily and cumulative volumes of ashfall are shown in Fig. 13. An eruption occurred from April 8th to April 21st, which continued until June 13th. During April 21st to June 3rd period the eruption rate increased. Ashfall amounts were not collected for the eight eruptions that took place on the 12th to the 25th of April. Instead we used the medium-classification average volume of 5000 t (Table 3) because the eruptions during this period were classified as “medium (Q3)” according to JMA. The peak volumes of the eruptions were clustered from May 6th to 23rd, 2008 (Fig. 13). The rate of eruption decreased in the next period from June 14th to July 28th. The pace of activity subsequently changed during August, with a single eruption every two or three weeks from the Showa crater; after October, there were a number of very small eruptions from the Minamidake crater. Using this method, we are able to reveal temporal change of the ashfall volumes, which is very important information in predicting the progress of the ongoing eruption.

5. Conclusion

The EAI provided a swift geometric method for assessing ashfall eruptions. In many cases, the distributions calculated using the EAI method correlated well with the observed data for the Usu and Asama volcanoes, in which small eruptions recently produced low columns. Under
these conditions, we could approximate the ashfall distribution as a single exponential function. The EAI method is useful for small, continuous eruptions and for small island volcanoes where terrestrial ashfall is naturally limited to the area of the island. When using the EAI method, it is important to determine the correct EAI calculation axis and to confirm the fit between several observation points and the calculated distribution.

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References

Fig. 13. Temporal changes of ashfall volumes determined by the EAI method. The column bars denote the daily ashfall volumes determined by the EAI method, and the step line represents the cumulative volume.


坂島火山における椭円近似による火山灰堆積量の推定法について

田島靖久・田村圭司・山越隆雄・津根 明・鶴本慎治郎

海に囲まれた火山島では観測できる場所が限られ、火山灰の堆積量を推定することが困難であった。また、火山灰が大量に降ることによって交通、健康、農作物へ影響を生じ、厚く堆積した斜面では土石流が発生しやすくなる。ゆえに火山灰の降下量（降灰量）や分布を迅速に把握する方法の開発は、火山学、防災学上の重要な研究対象となる。このため坂島のように海に囲まれ観測場所が限られる火山での迅速かつ少ない点から火山灰の堆積分布・量を推定する方法を検討した。我々は等層厚線が相似の椭円に近似されると仮定し、各点から得られる椭円近似した等層厚線の軸比を一定とし、分布を幾何学的に単純化した。また、降灰観測データの豊富な噴火事例を検証した結果、面積=層厚が $A = \alpha T^d$ （$T$: 層厚、$A$: 面積）とした場合、その減衰はほぼ $-1$ 乗に近似可能である。これらの関係より、火口位置などを椭円の軸端点とし、火山灰堆積分布に相当する分布軸が決まる場合、計算上2点の観測値から火山灰堆積量を推定することが可能となる。ただし、本手法では通常、計算軸を求める際に、計算に使用する2点以外の1〜4点程度の複数観測点の値が必要となる。本手法については分布軸が精度良く求められることと、複数の観測値を解析結果が矛盾なく説明できることを適応条件とした。本手法を用い坂島2008年の活動について60を超える噴火の火山灰堆積量を推定した。推定した分布から特定の場所における月ごとの累積降灰量を計算した結果は観測量を再現可能である。2008年の坂島の活動を日単位の堆積量として解析すると、ピークは5月6〜23日頃であったと推定される。本方法を適応することによって、これまで観測が難しかった坂島での火山灰堆積量観測が可能となる。